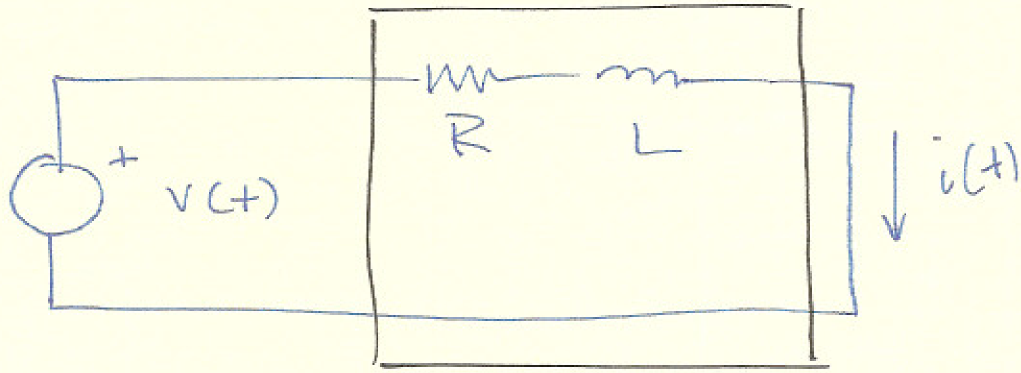
Ex: (cont..):



System

$$V(z) = (R+L)I(z) - (z^{-1} L I(z) + L i(-1)$$

$$i(-1) = 0$$

$h[n] \Leftarrow$ finished last class.

$V[n] = u[n]$

$$\therefore \quad V(z) = \frac{1}{1 - z^{-1}}$$

$$I(z) = V(z)H(z) = \frac{1}{(R+L)\left[1 - \frac{L}{R+L} z^{-1}\right]} * \frac{z}{z-1}$$

$$= \frac{B}{1 - \alpha z^{-1}} \cdot \frac{1}{1 - z^{-1}} \qquad B = \frac{1}{R+L}$$

$$\alpha = \frac{L}{R+L}$$

$$= \frac{A}{1 - \alpha z^{-1}} + \frac{B}{1 - z^{-1}}$$

$$A = \frac{-\beta\alpha}{1-\alpha}$$

$$B = \frac{\beta}{1-\alpha}$$

$$\therefore\ i[n] = \mathcal{Z}^{-1}\left[I(z)\right] = A\alpha^n u[n] + B\,u[n]$$

$$= \frac{\beta}{1-\alpha}\left[1 - \alpha^{n+1}\right] u[n]$$

Assuming exact reconstruction

$$\boxed{i(t) = \frac{\beta}{1-\alpha}\left[1 - \alpha^{t+T_s}\right], \quad t \geq 0}$$

corrisponds to 1 sample in discrete time domain. Assumed in last class.

Some Termonologies.

White Noise

It is a commonly used in random seq. The power spectrum of white noise is constant.

$$\phi_{xx}(e^{j\omega}) = \sigma_x^2$$

$$\therefore \quad \phi_{xx}[n] = \sigma_x^{22}\, \delta[n]$$

$\boxed{\sigma_x : \text{variance of sequence}}$

The avg. power of white noise signal is:

$$\phi_{xx}[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{xx}(e^{jw})\,dw$$

$$= \sigma_x^{\,2}$$

Infinite Impulse responce (IIR) system

A system for which the impulse responce is infinite in length (Recursive System)

$$h[n] = A\,\alpha^n\, u[n]$$

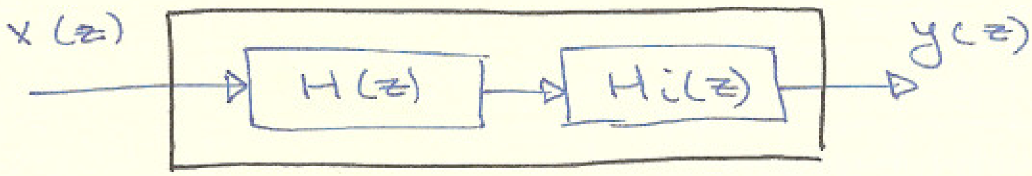
Finite Impulse responce (FIR) systen

$h[n]$ is finite in length. (non recursive System)

$$h[n] = A\alpha^n\,[u[n] - u[n-N]]$$

Inverse System

For a given LTI system with T/F $H(z)$ the corrisponding inverse system has a T/F $Hi(z)$ such that if both systems are cascaded, overall T/F will be $G(z) = H(z)\,Hi(z) \stackrel{!}{=} 1$.



$$G(z) = 1$$

$$g[n] = h[n] \circledast hi[n] = \delta[n]$$

Ex: Find the impulse responce of an inverse of the following system

$$H(z) = \frac{1 - 0.5z^{-1}}{1 - 0.9z^{-1}} \; ; \quad |z| > 0.9$$

SOL:

$$H_i(z) = \frac{1 - 0.9z^{-1}}{1 - 0.5z^{-1}} \; ; \quad |z| > 0.5$$

$$\therefore \; hi[n] = 0.5^n u[n] - 0.9 (0.5)^{n-1} u[n-1]$$

It is clear here that the system is IIR. because it goes on forever.

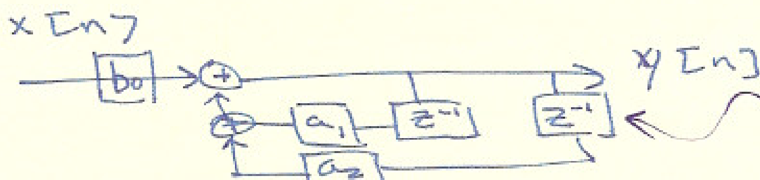
Block Diagram Repersentation of DE

(Implementation of LTI system defined by).

Ex:

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b\, x[n]$$

SOL:



delay function

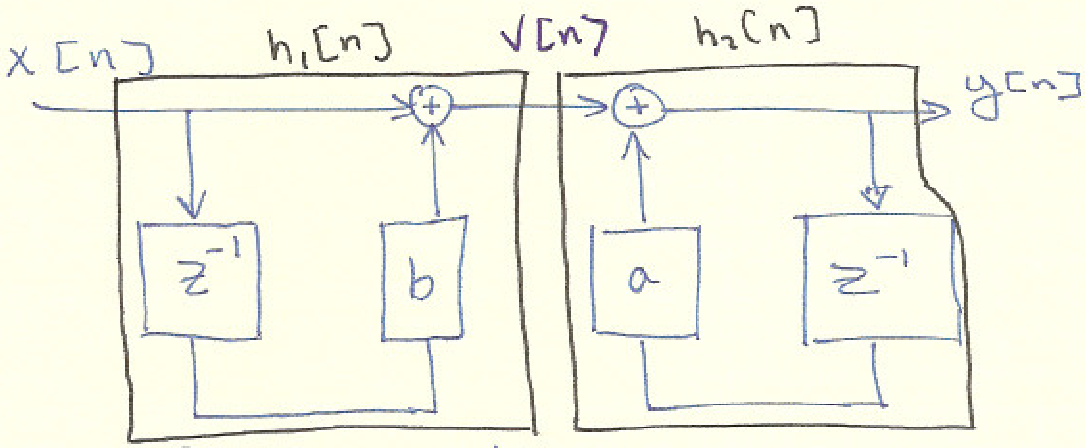
EX: 2 Implement the following system using block diagram.

$$y[n] = x[n] + b\,x[n-1] + a\,y[n-1]$$

SOL:

$$y[n] = v[n] + a\,y[n-1]$$

$$= x[n] + b\,x[n-1]$$

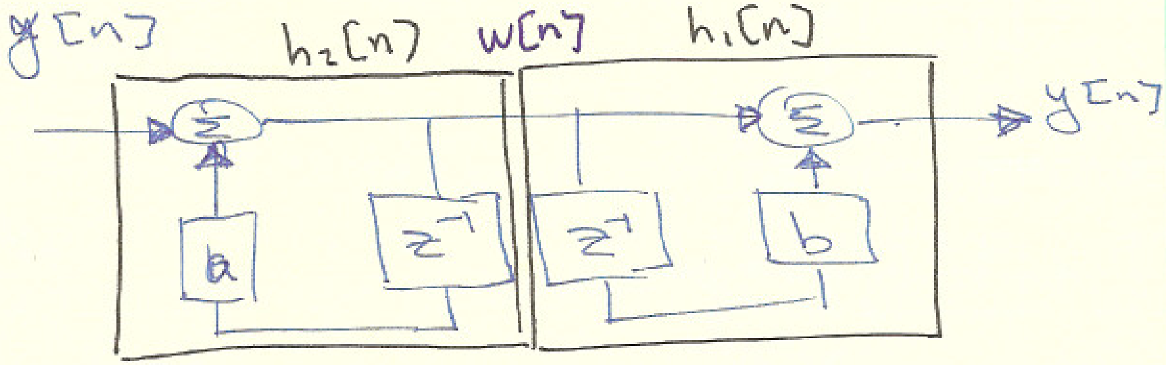


This type of block diagram is known as the Direct from implementation (D-I)

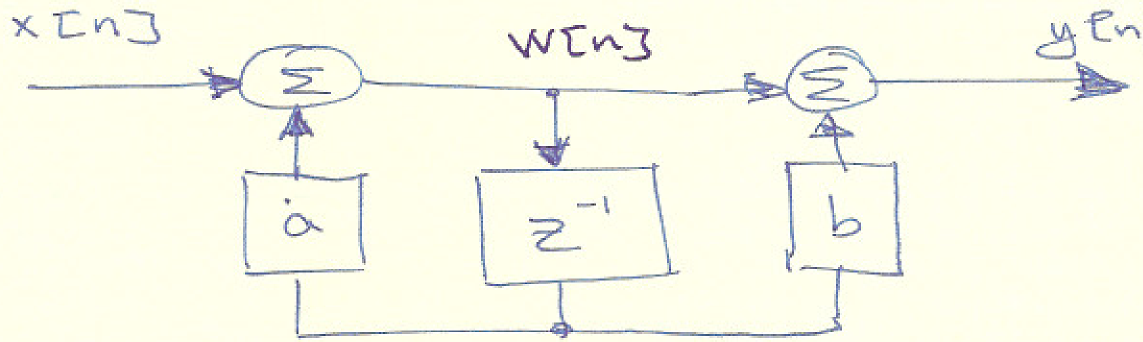
Using Commutetive property, the overall T/F

$$h[n] = h_1[n] \circledast h_2[n]$$
$$= h_2[n] \circledast h_1[n]$$

Therefore the alternate repersentation of the diagram can be drawn as.



The ~~preceeding~~ following figure is known as the direct form II.



$$w[n] = x[n] + a\,w[n-1]$$

$$y[n] = w[n] + b\,w[n-1]$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - a z^{-1}}$$

$$\frac{Y(z)}{W(z)} = 1 + b z^{-1}$$

$$\therefore \quad \frac{Y(z)}{X(z)} = \frac{1 + b z^{-1}}{1 - a z^{-1}}$$